Lesson 2.7: Negative Exponents and Reciprocals

Specific Outcome: 3.1 – Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$. 3.6 – Identify and correct errors in a simplification of an expression that involves powers. 3.3 – Apply the exponent laws to expressions with rational and variable bases and integral and rational exponents, and explain reasoning.

REVIEW (Gr. 9):

- Two numbers are **reciprocals** if their product is 1:

- Product of powers: $a^m \cdot a^n = a^{m+n}$
- $a^0 = 1$
- **NEW**: a power is never left with a negative exponent

Consider: 5^0 We know this power is equal to 1!

We can break down: $5^0 = 5^{2+-2} = 5^2 \cdot 5^{-2}$

We know: $5^0 = 1$

So then: $5^{-2} \cdot 5^2 = 1$

Therefore: 5^2 *and* 5^{-2} are ***reciprocals*** of each other because their product is 1.

And if follows: $5^{-2} = \frac{1}{5^2}$ *and* $5^2 = \frac{1}{5^{-2}}$

POWERS WITH NEGATIVE EXPONENTS

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n} \quad \textbf{and} \quad \frac{1}{a^{-n}} = \left(\frac{1}{a}\right)^{-n} = a^n \qquad a \neq 0$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \qquad a, b \neq 0$$

Practice: Write each power with a positive exponent.

a) 2^{-2} b) $(-5)^{-4}$ c) $\frac{1}{4^{-2}}$ d) $-\left(\frac{5}{3a}\right)^{-5}$ e) $(1.5)^{-3}$ f) $2x^{-3}$

Practice: Write each as a power with a positive exponent. Evaluate or simplify as applicable.

a) $\left(\frac{4}{9}\right)^{-2}$ b) $\frac{a}{4^{-2}}$ c) $\left(-\frac{x^3}{27}\right)^{-3}$ d) $(32)^{-2}$

HOMEWORK: P. 233 – #3, 4, 6, 7, 8(def)

COMBINE: NEGATIVE and RATIONAL EXPONENTS

$$a^{-\frac{m}{n}} = \left(\frac{1}{a}\right)^{\frac{m}{n}} \qquad \textbf{because exponent is negative}$$

$$= \left(\sqrt[n]{\frac{1}{a}}\right)^m \qquad \textbf{because exponent is a rational number}$$

Evaluate: $8^{-\frac{2}{3}}$

1. Write with a positive exponent: $8^{-\frac{2}{3}} =$

2. Write as a radical:

3. Simplify radical in brackets:

4. Evaluate:

Practice: Evaluate the following powers without a calculator where possible.

a) $49^{-\frac{3}{2}}$ b) $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$ c) $\left(-\frac{27}{8}\right)^{-\frac{2}{3}}$ d) $(0.81)^{-\frac{5}{2}}$ e) $\left(\frac{100}{9}\right)^{-1.5}$ f) $16^{-\frac{3}{4}}$

Problem Solving:

1. When the power $(-32)^{-0.4}$ is simplified to the form $\frac{a}{b}$, which one of the following is not a step in the simplification process?
   A. Changing the exponent to a positive value
   B. Finding the reciprocal of the exponent
   C. Converting the power to a radical
   D. Squaring a fraction

2. Calista found this solution for the following power. Circle her first mistake and write the correct solution.

$$\left(\frac{9}{25}\right)^{-\frac{5}{2}} = \left(\frac{25}{9}\right)^{\frac{2}{5}} = \left(\sqrt{\frac{25}{9}}\right)^{5} = \left(\frac{5}{3}\right)^{5} = \frac{3125}{243}$$

3.

Evaluate the following and arrange the answers from greatest to least.

Calculation 1. $-(27)^{-\frac{2}{3}}$ **Calculation 2.** $\left(\frac{1}{27}\right)^{\frac{1}{3}}$

Calculation 3. $(-27)^{\frac{2}{3}}$ **Calculation 4.** $\left(-\frac{1}{27}\right)^{-\frac{1}{3}}$

Place the calculation # with the greatest answer in the first box.
Place the calculation # with the second greatest answer in the second box.
Place the calculation # with the third greatest answer in the third box.
Place the calculation # with the smallest answer in the fourth box.

(Record your answer in the numerical response box from left to right)

3. Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155s^{\frac{5}{3}}f^{\frac{-7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, v is the speed in m/s, s is the distance between successive footprints of the same foot, and f is the foot length in m. Estimate the speed of the dinosaur, to the nearest tenth, when $s = 1$ *and* $f = 0.25$.

HOMEWORK: P. 233 – #9(dfh), 11, 12, 13(def), 15, 16, 19, 20