

Lesson 2.7: Negative Exponents and Reciprocals

Specific Outcome: 3.1 – Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$. 3.6 – Identify and correct errors in a simplification of an expression that involves powers. 3.3 – Apply the exponent laws to expressions with rational and variable bases and integral and rational exponents, and explain reasoning.

REVIEW (Gr. 9): Two numbers are **reciprocals** if their product is 1:

Similarly:

Practice: Write the reciprocal for each of the following numbers.

- a) 6 b) 11 c) -41 d) -0.5 e) $\frac{1}{2}$ f) $\frac{8}{5}$ g) $-\frac{4}{9}$ h) $-\frac{1}{7}$

POWERS WITH NEGATIVE EXPONENTS

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = \left(\frac{1}{a}\right)^{-n} = a^n \quad a \neq 0$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad a, b \neq 0$$

Practice: Write each power with a positive exponent.

- a) 2^{-2} b) $(-5)^{-4}$ c) $\frac{1}{4^{-2}}$ d) $\left(-\frac{1}{3}\right)^{-5}$ e) $\left(\frac{4a}{3}\right)^{-2}$ f) x^{-3} g) $\left(\frac{1}{x}\right)^{-7}$

HOMEWORK: P. 233 – 3, 5, 6, 7, 8

POWERS WITH NEGATIVE RATIONAL (Fractional) EXPONENTS

$$a^{-\frac{m}{n}} = \left(\frac{1}{a}\right)^{\frac{m}{n}} \quad \text{because exponent is negative}$$

$$= \left(\sqrt[n]{\frac{1}{a}}\right)^m \quad \text{because exponent is a rational number}$$

Consider: $8^{-\frac{2}{3}}$ We can evaluate this power in steps as follows:

1. Write with a positive exponent: $8^{-\frac{2}{3}} =$

2. Write as a radical (fractional exponent): $=$

3. Simplify radical in brackets: $=$

4. Evaluate: $=$

Practice: Evaluate the following powers by converting them to radicals.

a) $4^{-\frac{1}{2}}$

b) $49^{-\frac{3}{2}}$

c) $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$

d) $\left(-\frac{27}{8}\right)^{-\frac{2}{3}}$

e) $\left(\frac{1}{4}\right)^{-1.5}$

Problem Solving:

1. Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, v is the speed in m/s, s is the distance between successive footprints of the same foot, and f is the foot length in m. Estimate the speed of the dinosaur when $s = 1$ and $f = 0.25$.

2. Which expression is not equivalent to the others?

A. $a^{-\frac{4}{3}}$

B. $\left(\frac{1}{a^4}\right)^3$

C. $\left(\sqrt[3]{a}\right)^{-4}$

D. $\frac{1}{\sqrt[3]{a^4}}$

3. For all positive integers a and b , which of the following is not equivalent to $a^3\sqrt{b}$?

A. $a^3b^{\frac{1}{2}}$

B. $(a^6b)^{\frac{1}{2}}$

C. $\sqrt{a^6b}$

D. All of the expressions are equivalent to $a^3\sqrt{b}$.

*4. Evaluate the following and arrange the answers from greatest to least.

Calculation 1. $-(27)^{-\frac{2}{3}}$

Calculation 2. $\left(\frac{1}{27}\right)^{\frac{1}{3}}$

Calculation 3. $(-27)^{\frac{2}{3}}$

Calculation 4. $\left(-\frac{1}{27}\right)^{-\frac{1}{3}}$

Place the calculation # with the greatest answer in the first box.

Place the calculation # with the second greatest answer in the second box.

Place the calculation # with the third greatest answer in the third box.

Place the calculation # with the smallest answer in the fourth box.

(Record your answer in the numerical response box from left to right)

--	--	--	--