Lesson 2.6: Fractional Exponents and Radicals

Specific Outcome: 3.3 – Apply the exponent laws to expressions with rational and variable bases and integral and rational exponents, and explain reasoning. 3.4 – Express powers with rational exponents as radicals and vice versa. 3.2 – Explain, using patterns, why $a^{1/n} = \sqrt[n]{a},\ n > 0$. 3.6 – Identify and correct errors in a simplification of an expression that involves powers.

POWERS (Gr. 9 Review)

A power is a number written with a ***base*** and an ***exponent:*** a^n ← ***Exponent*** (***Base*** = a)

- A base with no exponent is understood to have an exponent of 1: so, $5 = 5^1$
- Any power with a non-zero base and an *exponent of 0* is equal to 1: so, $3^0 = 1,\ x^0 = 1$
- When a power has a *negative* or *fractional* base, the base must be enclosed in brackets! Eg. $(-6)^4$ $\left(\frac{2}{3}\right)^4$

Practice: State the base and the exponent in each of the following powers.

a) 8^3 b) $(-10)^7$ c) $\left(\frac{4}{6}\right)^2$ d) x^y

Practice: Evaluate each power as a decimal, to the nearest hundredth, using a calculator

a) $1000^{\frac{1}{3}}$ b) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$ c) $0.25^{\frac{1}{2}}$

Now evaluate each of these radicals using a calculator.

a) $\sqrt[3]{1000}$ b) $\sqrt[4]{\frac{16}{81}}$ c) $\sqrt[2]{0.25}$

What conclusion can you make about a)'s, b)'s and c)'s above?

POWERS WITH FRACTIONAL EXPONENTS WITH *NUMERATOR 1*

- When $n \in N$ and $a \in Q$,

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Practice: Write each power as a radical, then evaluate if possible.

a) $27^{\frac{1}{3}}$ b) $49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ e) $25^{0.5}$ f) $x^{\frac{1}{5}}$

Practice: Write each radical as a power. Use brackets as necessary.

a) $\sqrt{29}$ b) $\sqrt[3]{73}$ c) $\sqrt[5]{-122}$ d) $\sqrt[4]{\frac{6}{7}}$ e) $\sqrt{-\frac{3}{4}}$ f) $\sqrt[3]{x}$

HOMEWORK: P. 227 – 3, 4(ab), 5, 6, 7(ace)

POWERS WITH FRACTIONAL EXPONENTS

When $\boldsymbol{m,n} \in N$, and $\boldsymbol{a} \in Q$,

$$\boldsymbol{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m} \qquad \boldsymbol{OR} \qquad \boldsymbol{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$$

Write $40^{\frac{2}{3}}$ as a radical in 2 ways:

Practice: Write the powers in radical form. Evaluate starred ones.

*a) $27^{\frac{4}{3}}$ b) $(-35)^{\frac{3}{5}}$ *c) $81^{\frac{5}{4}}$ *d) $\left(\frac{9}{4}\right)^{1.5}$ e) $(-x)^{\frac{3}{5}}$

Practice: Write each radical in exponential form.

a) $\sqrt[4]{19^3}$ b) $\left(\sqrt[3]{-42}\right)^2$ c) $\left(\sqrt{200x}\right)^5$ d) $\left(\sqrt[3]{\frac{-8}{27}}\right)^4$ e) $\sqrt{83x^7}$

Problem Solving:

1. A cube has a volume of 30 cm^3. Write the side length of the cube as:
   a) a radical
   b) a power

2. Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kg, of a mammal with a body mass of m kg. Estimate the brain mass of each animal. Round to the nearest hundredth if necessary.
   a) a moose with a body mass of 512 kg
   b) a cat with a body mass of 5 kg

   c) Given $1\ lb. = 0.454\ kg$, determine the answer from b) in pounds, to the nearest hundredth.

HOMEWORK: P. 227 – 8, 9, 10-13(abc), 17-19